Single-shot time-reversed optical focusing into and through scattering media: Supporting Information

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Abstract: This document provides supporting information to the manuscript entitled "Single-shot time-reversed optical focusing into and through scattering media". It includes a full mathematical description of the proposed single-shot optical time-reversal method and supplementary figures to illustrate the method.

Contents:

S1: Mathematical description of the proposed single-shot optical time-reversal method

S2: Supplementary figure

5 pages, 1 figure.

S1. Mathematical description of the proposed single-shot optical time-reversal method

Without loss of generality, we assume that the first element of the incident optical field on the scattering medium is A_0, while all the other elements are zeros. That is, we consider an incident field having one input mode, which can be described as

$$\mathbf{E_{in}} = (A_0 \quad 0 \quad \cdots \quad 0)^T{}_{1\times J}, \tag{S1}$$

where "T" represents the transpose operator and J is the total number of vector elements. When $\mathbf{E_{in}}$ passes through a scattering medium described by a transmission matrix $\mathbf{T} = (t_{mj})_{M\times J}$, whose element t_{mj} follows a circular Gaussian distribution, the scattered light field has the following expression:

$$\mathbf{E_s} = \mathbf{T}\cdot\mathbf{E_{in}} = A_0 \cdot (\mathrm{t}_{11} \quad \mathrm{t}_{21} \quad \cdots \quad t_{M1})^T{}_{1\times M}. \tag{S2}$$

$\mathbf{E_s}$ will interfere with a plane reference beam $\mathbf{E}_R = A_R \exp(i\varphi_R)$, where A_R and φ_R are the constant amplitude and phase of $\mathbf{E}_R$. The resulting hologram can be modeled as

$$I = \left|\mathbf{E_s} + \mathbf{E_R}\right|^2 = (\mathbf{E_s} + \mathbf{E_R})(\mathbf{E_s} + \mathbf{E_R})^*. \tag{S3}$$

Obviously, the m-th element of the hologram is

$$I^{(m)} = A_R{}^2 + A_0{}^2\left|t_{m1}\right|^2 + 2A_R A_0\left|t_{m1}\right|\cos(\varphi_{m1} - \varphi_R). \tag{S4}$$

In Eq. (S4), we imply that $t_{mj} = \left|t_{mj}\right|\exp(i\varphi_{mj})$.

In the playback step, the reference beam is modulated by an SLM with a phase pattern ϕ. After propagating through the scattering medium, the optical field of the time-reversed focus becomes

$$\mathbf{E_f} = \mathbf{T}^T A_R \exp(i\varphi_R + i\phi). \tag{S5}$$

The j-th element of $\mathbf{E_f}$ is

$$\mathbf{E_f}^{(j)} = \sum_{m=1}^{M}\left|t_{mj}\right| A_R \exp[i(\varphi_{mj} + \varphi_R + \phi^{(m)})], \tag{S6}$$

where $\phi^{(m)}$ is the m-th element of ϕ.

For Mode 1 and Mode 2 of the described single-shot optical time-reversal method, $\phi^{(m)}$ is directly determined according to the intensity of the hologram. That is,

$$\phi^{(m)} = 2\pi I^{(m)} / \max(I), \tag{S7}$$

for Mode 1, and

$$\phi^{(m)} = 2\pi[I^{(m)} - mean(I)] / \max\{abs(I - mean(I))\}, \tag{S8}$$

for Mode 2, where $mean(\cdot)$ denotes the average value over all the pixels of the hologram. These two modes are inspired by analog time-reversal approaches, where the conjugated field can be generated by reading an intensity hologram recorded in the volume of a crystal. At the focus position, where $j=1$, the phase terms in Eq. (S6) are not uniformly distributed any more when $\phi^{(m)}$ in Eq. (S7) or Eq. (S8) is added. Instead, more phasors will gather together and finally form the constructively interfered focus [See Fig. S1(a1) and Fig. S1(b1)]. However, at positions where $j \neq 1$, the added phase term $\phi^{(m)}$ does not change the statistical distribution of the total phase of $\mathbf{E_f}^{(j)}$ because $\phi^{(m)}$ and φ_{mj} ($j \neq 1$) are independent in this case. Thus, the phase terms in Eq. (S6) are still uniformly distributed [See Fig. S1(a2) and Fig. S1(b2)], and a speckle background is formed at that position.

In Mode 3 of the described single-shot optical time-reversal method, we first subtract the average value of the hologram. Because the phase term $\varphi_{m1} - \varphi_R$ in Eq. (S4) is uniformly distributed among [0, 2π] for all the elements of the hologram, we have

$$mean(I) = A_R{}^2 + A_0{}^2 \cdot mean(|t_{m1}|^2). \tag{S9}$$

Therefore, element m of the average-subtracted hologram is

$$\begin{aligned} \Delta I^{(m)} &= I^{(m)} - mean(I) \\ &= A_0{}^2[|t_{m1}|^2 - mean(|t_{m1}|^2)] + 2A_R A_0 |t_{m1}| \cos(\varphi_{m1} - \varphi_R) \\ &\approx 2A_R A_0 |t_{m1}| \cos(\varphi_{m1} - \varphi_R). \end{aligned} \tag{S10}$$

Because $|t_{m1}|$ is much less than unity, the first term in the middle line of Eq. (S10) is negligible if the cosine term is not too close to zero, the violation of which occurs only with a low probability for a fully developed speckle field. When $\Delta I^{(m)}$ >0, we can infer from Eq. (S10) that φ_{m1} is between $[-\pi/2+\varphi_R, \pi/2+\varphi_R]$. In this case, we set $\phi^{(m)}=0$. Otherwise, φ_{m1} should be between

$[\pi/2+\varphi_R, 3\pi/2+\varphi_R]$. In this case, we set $\phi^{(m)}=\pi$. Inserting the $\phi^{(m)}$ here back into Eq. (S6), we can find that the first element of the time-reversed field $\mathbf{E_f}$ is

$$\mathbf{E_f}^{(1)} = \sum_{m=1}^{M} \left|t_{m1}\right| A_R \exp[i(\varphi_{m1}+\varphi_R+\begin{cases}0, if \quad \Delta I^{(m)}>0 \\ \pi, otherwise\end{cases})]. \tag{S11}$$

Obviously, the phase terms of all the phasors in the summation are in the range of $[-\pi/2+2\varphi_R, \pi/2+2\varphi_R]$. Thus these phasors will interfere constructively, which forms the time-reversed focus. For other elements where $j \neq 1$, the phase terms are uniformly distributed between $[0, 2\pi]$, thus forming a speckle background as a result of incoherent addition [See Fig. S1(c1) and Fig. S1(c2)].

From the above derivation, we can see that by simply subtracting the average of a single hologram, we can divide the phases of speckles into two groups according to whether they contribute to constructive interference or not. This division is surprisingly effective and does not require the precondition that the intensity of signal beam must be far smaller than the intensity of the reference beam, as required by the existing quasi-single-shot method, because the average subtraction can be considered as an adaptive process for the hologram, as illustrated in Eqs. (S10-S11). This advantage makes single-shot time-reversed ultrasonically encoded (TRUE) optical focusing possible where the intensity of the ultrasonically untagged photons is strong.

S2. Supplementary figure

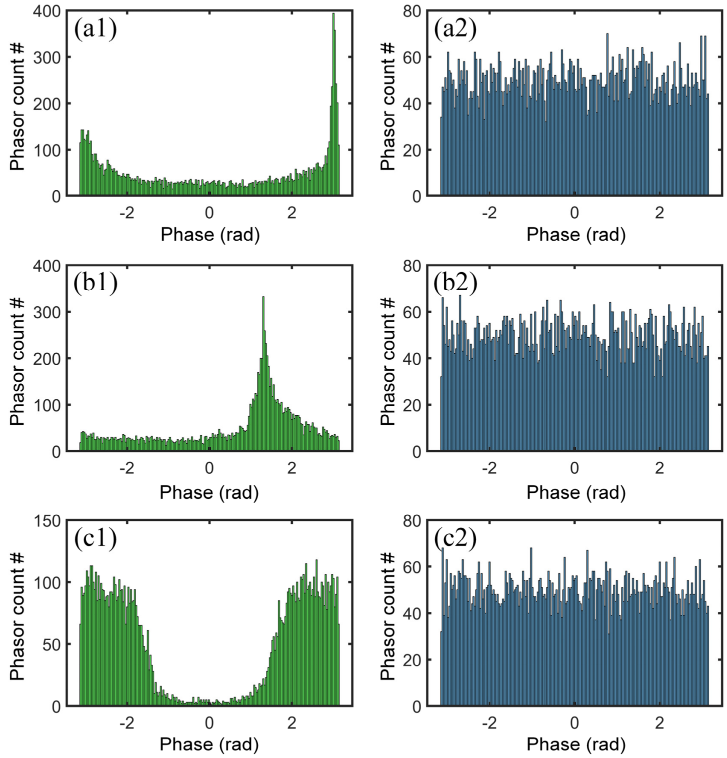

Fig. S1. Examples of the statistical histograms of the phase terms in Eq. (S6) for the three modes of the described single-shot optical time-reversal method. (a1), (b1), and (c1) are for $j=1$ (i.e., at a focal position) for Mode 1, Mode 2 and Mode 3 respectively, while (a2), (b2), and (c2) are for $j \neq 1$ (i.e., at a speckle background position). We can see that, the added phase term $\phi^{(m)}$ will make more phasors align to interfere constructively at the focal position. At non-focal positions, the phases of the phasors are still uniformly distributed, thus forming a speckle background as a result of incoherent addition. Among these three modes, Mode 3 has the best ability for phase alignment because most phasors are adaptively aligned within the angular range $[-\pi,-\pi/2]\cup[\pi/2,\pi]$, as illustrated in the mathematical description above.